AIAA 82-4182

Comparison of Rezone Algorithms for Incompressible Fluid Flow Calculations

M. M. Sussman*
Westinghouse Electric Corporation, West Mifflin, Pa.

Abstract

THE art of computational fluid dynamics has progressed to the point that computations can be performed using mesh grids that conform with features of the flow solution or that change as the solution domain changes. Such cases include oscillating airfoils and other flow-induced motions of solid bodies, where the locations of essential features can move sinusoidally from place to place. It is the purpose of this paper to present a comparison of two algorithms designed to account for the effects of temporally changing meshes in order to determine the one with the lesser adverse effect on the fluid flow solution.

Contents

The conservation of momentum (Navier-Stokes) equations can be formulated so that motion of mesh points is treated naturally in the convection term. This fact has been exploited in formulating the ICED-ALE finite difference method of Hirt et al.[1] and also the quasi-Eulerian finite element method as used by Belytschko and Kennedy,[2] to name just two examples. In this synoptic, numerical experiments are described which indicate that this natural approach based on conservation of momentum may not be the most accurate one.

In the following paragraphs, two approaches are briefly described—the first based on conservation of momentum as mentioned above and the second on biquadratic interpolation. Then three different flow configurations are described and comparison of the "rezone algorithms" for these configurations are given. The conclusion reached is that the rezone algorithm based on interpolation is to be preferred over the one based on conservation of momentum.

The numerical experiments described below were performed using a computer program based on the ICED-ALE approach of Hirt et al.[1] The primary differences between the program used and that of Ref. 1 are that the convection terms are evaluated before the new pressure values are found and that a direct, rather than iterative, solution method is used to find the pressure solution.

In order to discuss the specific forms of the rezone algorithms which were investigated, let $x^{(n)}$, $u^{(n)}$, and $\rho u^{(n)}$ denote position, velocity, and momentum vectors, respectively, at the end of the nth time step. Here, ρ denotes the constant value of density. Let $x^{(n+1)}$ denote the new mesh location and u' and $\rho u'$ the changed velocity and momentum values at $x^{(n+1)}$ based on the values $u^{(n)}$. One rezone scheme is based on the notion that the change in momentum inside the control volume $V^{(n)}$ is given by the amount of momentum swept through its walls as they move from their old to their new positions. Hence $\rho u'$ can be found by approximating the expression

$$\int\int_{V^{(n)}} (\rho u' - \rho u^{(n)})\,\mathrm{d}A$$

$$= -\oint_{\partial V^{(n)}} \rho u^{(n)} [(x^{(n+1)} - x^{(n)})\cdot n]\,\mathrm{d}\ell$$

where n denotes the unit outward normal. The approximation used is the same as that used in the convection terms save that central differences rather than partial donor cell differences are employed.

A second rezone algorithm is given by isoparametric biquadratic interpolation. If the point $x^{(n)}$ and its eight nearest neighbors are regarded as a nine-node biquadratic finite element, then u' can be found from $x^{(n+1)}$ in the manner described, for example, by Ergatoudis et al.[3]

Two of the flow configurations used in the tests are Jeffrey-Hamel flows. Jeffrey-Hamel flows are self-similar flows in diffusers or nozzles. That is, when expressed in polar (r,θ) coordinates, the angular velocity vanishes and the radial velocity is given by $u_r = f(\theta)/r$ for certain functions $f(\theta)$ (see Ref. 4 for more detail). The functions for the two cases are sketched in Fig. 1. Case 1 involves a diffusion-dominated flow at a Reynolds number of 1000 with a region of reversed flow. Case 2 involves a convection-dominated inwardly directed flow at a Reynolds number of 5000.

The reference grid for case 1 is a uniform 10×10 grid. That for case 2 is a graded 19×19 grid with finer spacing in the boundary layer. In both cases the interior mesh points were moved up and down in a sinusoidal (in time) fashion. Figure 2 shows an example of a distorted mesh for case 2. At each time step the root-(spatial) mean-square (rms) error was computed. These errors fluctuate sinusoidally and their minimum and maximum amplitudes expressed relative to the solution are given in Table 1.

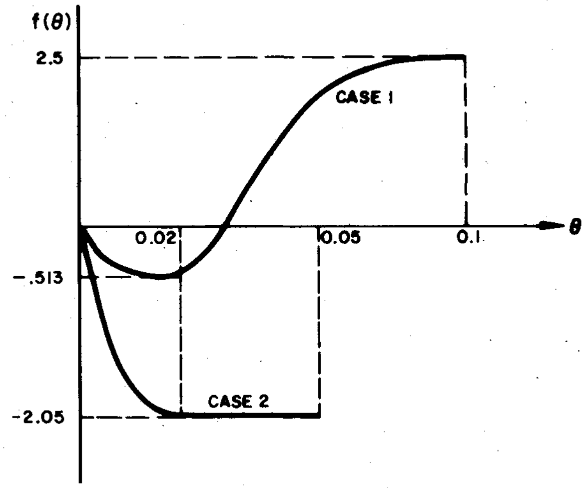

Fig. 1 Jeffrey-Hamel functions for cases 1 and 2.

Received June 23, 1981; synoptic received Nov. 16, 1981. This paper is declared a work of the U. S. Government and therefore is in the public domain. Full paper available from National Technical Information Service, Springfield, Va. 22151, at the standard price (available upon request).

*Senior Mathematician. Member AIAA.

Table 1 Fluctuating rms errors for rezone algorithms in Jeffrey-Hamel flow

		No rezone algorithm, %		Momentum conservation, %		Biquadratic isoparametric interpolation, %	
		min	max	min	max	min	max
Case 1	Pressure	100	100	70	100	32	46
	Velocity	2.8	9.8	8.6	19.7	3.7	4.4
Case 2	Pressure	6.1	82	16.9	96	2.6	5.0
	Velocity	1.1	2.9	5.8	19.3	0.8	1.0

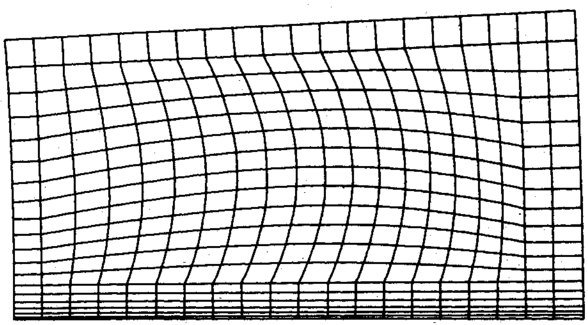

Fig. 2 Distorted mesh for Jeffrey-Hamel flow, case 2.

Table 2 Capture ratios for different frequencies of grid motion with natural vortex-shedding frequency of 0.5410 Hz

Frequency of grid motion	No rezone algorithm	Momentum conservation	Biquadratic isoparametric interpolation
0.3607	4.7	25.0	1.7
0.4058	26.0	No data	6.3
0.4509	196.0	75.0	22.0
0.6312	384.0	400.0	0.61
0.6763	108.0	21.0	0.44
0.7214	4.4	6.5	0.37

The first columns of Table 1 display the results of experiments performed with moving grid points but with no rezone strategy at all; i.e., velocities are carried from the end of one time step to the beginning of the next without regard for the effects of grid motion. For the problems in Table 1, the superiority of biquadratic isoparametric interpolation is clear. It is surprising that rezone based on conservation of momentum fares less well than no rezone at all. It is also surprising that, despite the interdependence of pressure and velocity, in some cases the pressure solution is entirely erroneous (100% error) while the velocity solution remains reasonably accurate.

The third flow configuration for which comparisons were made is two-dimensional confined flow past a rectangular obstacle. Such flow exhibits a characteristic "vortex shedding" behavior similar to a von Kármán vortex street. The natural frequency of the vortex shedding is extremely sensitive to the presence of a periodic driving force such as forced motion of the obstacle. If a driving force is present at a frequency within about 20% of the natural vortex shedding frequency, that natural frequency will be suppressed in favor of the driving frequency and it is impossible to observe the existence of a natural frequency. This "lock-in" phenomenon can also be reproduced in computer simulations, as has recently been studied by Hurlbut et al.,[5] among others. For the present study, it was found that even nonphysical mesh motions can induce lock-in. The degree of lock-in to the nonphysical mesh motion forms the basis for comparison of the two rezone schemes, as described below.

The problem used for testing is a rectangular duct 1.3 cm wide with a 0.5 cm wide × 0.4 cm long obstacle offset slightly from the centerline. The mesh used consists of square cells. Inlet velocity is uniformly 1 cm/s and Reynolds number is 1300 based on duct width. The most sensitive indicator of vortex shedding proves to be the transverse velocity just downstream of the center of the obstacle. In the undriven case, this velocity oscillates as a pure sine wave at a frequency of 0.5410 Hz. For the tests, a transverse sinusoidal motion was imposed on the grid points in a rectangle starting 0.3 cm downstream of the obstacle and coming within 0.3 cm of the duct walls.

The boundaries in the problem described above are stationary so that the mesh motion has no physical counterpart. In principle, use of a perfect rezone algorithm would leave the natural vortex-shedding frequency unaltered. With this in mind, a greater tendency to lock-in indicates the less accurate rezone algorithm. Both rezone algorithms showed a tendency to lock-in. This tendency can be measured by the "capture ratio," which is defined as

$$CR = (U_{\text{grid frequency}}) / (U_{\text{natural frequency}})$$

where U denotes the amplitude (at the indicated frequency) of the Fourier transform of the velocity mentioned above. A large capture ratio indicates suppression of the physically natural frequency in favor of the frequency of mesh motion. The results summarized in Table 2 support those of Table 1; namely, that isoparametric biquadratic interpolation provides a more accurate rezone scheme than the one based on momentum conservation.

References

[1] Hirt, C. W., Amsden, A. A., and Cook, J. L., "An Arbitrary Lagrangian-Eulerian Computing Method for All Flow Speeds," *Journal of Computational Physics*, Vol. 14, March 1974, pp. 227-253.

[2] Belytschko, T. and Kennedy, J. M., "Computer Models for Subassembly Simulation," *Nuclear Engineering and Design*, Vol. 49, March 1974, pp. 17-38.

[3] Ergatoudis, I., Irons, B. M., and Zienkiewicz, O. C., "Curved, Isoparametric, 'Quadrilateral' Elements for Finite Element Analysis," *International Journal of Solids and Structures*, Vol. 4, Jan. 1968, pp. 31-42.

[4] Lu, P.-C., *Introduction to the Mechanics of Viscous Flows*, Hemisphere Publishing Co., Washington, 1977.

[5] Hurlbut, S. E., Spaulding, M. L., and White, F. M., "Numerical Solution of the Time Dependent Navier-Stokes Equations in the Presence of an Oscillating Cylinder," *Proceedings on Symposium of Nonsteady Fluid Dynamics*, American Society of Mechanical Engineers, 1978, pp. 201-206.